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An experimental investigation of natural convection in a horizontal cylinder

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This work deals with an experimental investigation of natural convection inside a horizontal cylinder. The fluid, geometry, and thermal boundary condition were chosen so as to have a high Prandtl number and unit-order Grashof number.

The thermal boundary condition was established by imposing temperatures at two points, 180° apart, on the circumference of the cylinder. The resulting boundary condition for the full 360° was found experimentally and is presented. The apparatus was constructed so that the entire cylinder could be rotated in order to introduce an arbitrary heating angle into the boundary condition.

Temperature profiles and streamline patterns were observed at steady state for various values of this heating angle and for various initial conditions. For some cases, velocity profiles were also plotted. It was found that the interior 'core' of fluid was thermally stratified when the diameter containing the two imposed temperatures was horizontal. The flow occurred primarily in the region close to the cylinder wall. The cylinder was rotated so that the diameter containing the two imposed temperatures made an angle with the horizontal. The hotter of the imposed temperatures was always below the horizontal diameter of the cylinder. As this angle of rotation increased, it was found that the velocities encountered in the fluid increased and the degree of thermal stratification in the core region decreased. It was also found that the steady-state results were identical for the different initial conditions imposed. The results of this study are compared with previous work, both analytic and experimental.

1. Introduction

The problem presented here considers the natural convection of fluid in a completely confined horizontal cylinder. This particular configuration was chosen because the circular geometry has no stagnation points, a phase angle is easily included in the study, and there is adequate previous work with which to compare.

The first work dealing with this high Rayleigh number, internal convection problem, is attributed to Ostrach (1950). He considered the problem of a high Grashof number and unit-order Prandtl number. (The Rayleigh number is equal to the product of the Grashof and Prandtl numbers. Hence, a high Rayleigh number can correspond to a high Grashof number and unit-order Prandtl number or a unit-order Grashof number and high Prandtl number.) Ostrach considered a cosine temperature distribution around the walls of the cylinder with the temperature extrema located along the horizontal diameter, the condition referred to as 'heated from the side'. Ostrach predicted that both thermal and viscous boundary layers existed near the walls of the cylinder after postulating that the interior core was isothermal and rotated as a solid body. Martini & Churchill (1960) experimented with air in a horizontal cylinder. Their boundary condition was a step function with each half of the cylinder maintained at a different temperature. Heating was directed from the side. They observed that the flow took place in a boundary layer near the cylinder wall while the core region was essentially stagnant and thermally stratified with the temperature increasing vertically upward. This work cast doubt as to the existence of an isothermal rotating core when heating is directed from the side.

Weinbaum (1964) studied the identical problem as Ostrach (1950), but introduced a phase angle so that situations ranging from heating from the side (phase angle of zero) to heating from below (phase angle of 90°) could be studied. Ostrach (1968) and Ostrach & Menold (1968) investigated the same configuration as Weinbuam but considered the case of high Prandtl number and unit-order Grashof number. They found that an isothermal rotating core occurred only when the phase angle, ϕ, was in the range of $20°–30° < \phi < 90°$. Sabzevari & Ostrach (1966) experimented with the high Prandtl number, unit-order Grashof number situation and the cosine boundary condition. They presented velocity and temperature profiles along various radii of the cylinder. Their conclusions show the dependence of the flow on the heating angle, ϕ. For $\phi = 0°$, they conclude that the core is stagnant and thermally stratified, thus agreeing with Martini & Churchill (1960). For $\phi = 15°$, they say that the core does not act as a single cell. For $30° < \phi < 90°$, they observe the isothermal rotating core. Hantman & Ostrach (1968) analyzed the same problem as Ostrach (1968) and predicted that in the neighbourhood of $\phi = 0$, the core would be thermally stratified and the streamlines horizontal. They further predicted that the core will be isothermal only when the phase angle is greater than the lower limit which was indicated earlier.

A somewhat similar problem to the circular cylinder is the natural convection in a rectangular cavity. Pillow (1952) and Batchelor (1954) were the first to study this problem analytically. Pillow considered heating from below, while Batchelor discussed heating from the side. Eckert & Carlson (1961) performed experiments for the heating from the side configuration, while Wilkes (1963) and de Vahl Davis (1967) did numerical studies on the problem. All except Pillow agreed on a thermally stratified core with horizontal streamlines. Elder (1965) performed additional experiments for the heating from the side case. At high Rayleigh numbers he observed secondary and tertiary flows. Elder (1966) later did numerical work which agreed with his experiments. Gill (1966) did the analytic counterpart of Elder's experiments. Rossby (1965) experimented with the rectangular geometry non-uniformly heated from below and Somerville (1967) did a numerical study corresponding to Rossby's experiments. The asymmetry inherent in this problem is apparent in the results.

Although the circular and rectangular geometries represent somewhat related problems, there are some important distinctions between them. The rectangular geometry contains stagnation points at the corners. No such problems are encountered with the circular geometry. A heating angle is readily included in the study with the circular geometry. This is not so for the rectangular geometry. Most of the rectangular studies involve heating from the side only. The case of heating from below (the only other heating configuration studied with the rectangular geometry) will later be shown to include a thermal instability which makes the two-dimensional results questionable.

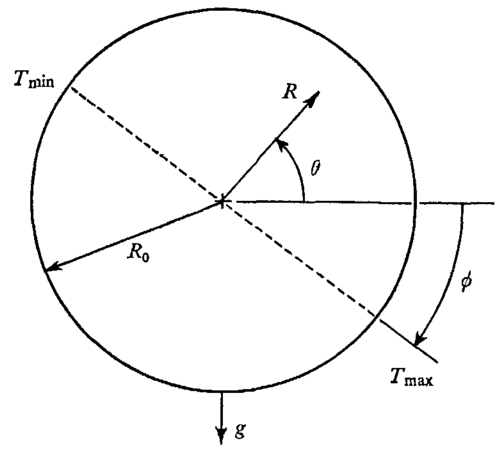

FIGURE 1. The geometry.

In the present work, a scheme by which streamlines can be traced is tested and used. More precise knowledge of the intricacies of the fluid motion in the core is made available than was accessible from velocity profiles alone. This method was used for various values of the heating angle, ϕ (see figure 1) to gain more insight into the different flow situations noted by Sabzevari & Ostrach (1966) and to investigate the dependence of the flow on the heating angle.

Secondly, knowledge as to the effect of the initial condition is sought. The analytic work so far treats only the steady-state solution with no mention of the initial condition. The question of the uniqueness of the steady-state solution is now being raised. If different initial conditions are utilized, will the steady-state result be the same? Is there any way in which the isothermal rotating core can be made to persist when heating is directed from the side?

Hellums & Churchill (1962) considered the unsteady terms in a numerical solution using the step-function boundary condition of Martini & Churchill (1960) with the fluid initially at rest and at a uniform temperature equal to the average of the imposed wall temperatures. Wilkes (1963) includes some transient solutions in his numerical analysis of the rectangular geometry. However, nowhere is the problem studied with a variety of initial conditions.

2. Experimental equipment and procedure

The test cylinder was made from a length of copper pipe. It is 8 in. long, has an inside diameter of 5 in. and a wall thickness of $\frac{1}{4}$ in. These dimensions were chosen for a number of reasons. First, the cylinder had to be long enough so that end effects would be negligible. This is of utmost importance if these results are to truly characterize a two-dimensional problem. Secondly, the length had to be such that the photographic equipment could focus accurately on the mid-plane of the cylinder. This is essential for the scheme employed to trace stream-lines and plot velocity profiles. The two-dimensionality of the results was verified experimentally as will be described subsequently.

The ends of the cylinder were sealed off with clear plexiglass windows and the entire cylinder was supported on smooth roller bearings. Two circular slots, $\frac{1}{8}$ in. deep and 180° apart, were milled axially into the cylinder wall. Two heat exchangers, constructed from $\frac{3}{4}$ in. diameter copper pipe, were soldered into these slots. Distilled water at a constant temperature was circulated through one heat exchanger while distilled water at another constant temperature was circulated

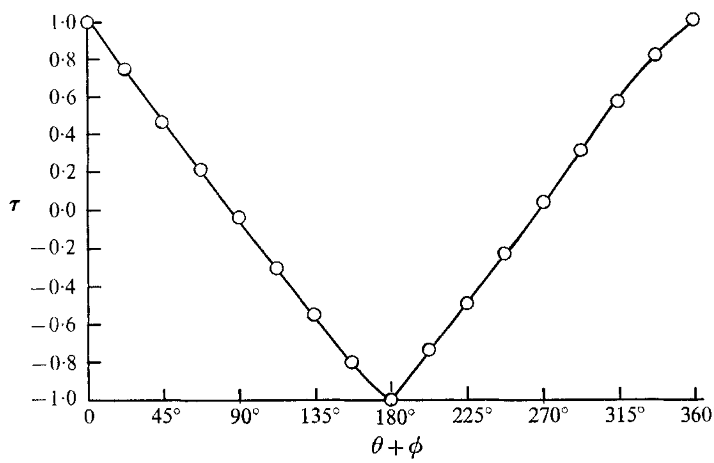

FIGURE 2. The thermal boundary condition.

through the other. In this way, T_{max} equal to 76·1 °F and T_{min} equal to 68·6 °F were imposed on two diametrically opposed points on the inside wall of the cylinder. This determined T_0, defined at $\frac{1}{2}(T_{max}+T_{min})$, and ΔT, defined as $\frac{1}{2}(T_{max}-T_{min})$, to be 72·35 and 3·75 °F respectively. The water temperature for each heat exchanger was controlled to within $\pm$ 0·01 °F by means of a Haake, Model Ne, constant temperature circulator.

The temperature distribution on the interior wall of the cylinder was measured by 24 B & S gauge copper-constantan thermocouples. The circumferential temperature distribution at the mid-plane of the cylinder was measured by 16 of the thermocouples, located every $22\frac{1}{2}$° around the perimeter. The remaining 8 thermocouples were placed 1 and 2 in. on either side of the mid-plane along two diametrically opposed axial lines and were used to detect any axial temperature gradient. The e.m.f.'s were measured using a Leeds and Northrup, Type 8686 millivolt potentiometer. This instrument can be read to an accuracy of 1 μV. It was found that the temperature measurements were accurate to within 0·1 °F. This represents an error of less than 2·7 %.

The resulting wall temperature distribution, found experimentally, is shown in figure 2. It should be noted that this boundary condition curve is not perfectly symmetrical. For $0° < \theta+\phi < 180°$, the curve for τ defined as $(T-T_0)/\Delta T$ is approximately 0·038 lower than the mirror image of the curve for $180° < \theta+\phi < 360°$. However, it was found experimentally that this boundary condition remained constant even if the heating angle was changed after fluid motion had begun. This is a necessity for the part of the work dealing with changes of the initial condition. Although this boundary condition is not the same as the cosine distribution used by Sabzevari & Ostrach (1966), it is expected that the qualitative behaviour should be similar.

The entire experiment was carried out in a temperature-controlled room. The temperature in the room was always kept at the average of the two imposed temperatures, T_0.

The working fluid was a silicone oil manufactured by the General Electric Company. It is known commercially as SF-96 (2000) and has a nominal kinematic viscosity of 2000 centistokes at 77 °F. The various properties of the fluid are supplied by the General Electric Company. Note should be taken of the variation of viscosity with temperature. The average temperature, T_0, was used to determine the viscosity. With knowledge of the fluid properties, ΔT, and the cylinder geometry, the values of the dimensionless parameters were calculated to be: Prandtl number, $Pr = 19{,}950$; Grashof number, $Gr = 1{\cdot}10$; and Rayleigh number, $Ra = 21{,}950$, where $Pr = \nu/\kappa$, $Gr = g\beta\Delta T R_0^3/\nu^2$ and $Ra = g\beta\Delta T R_0^3/\nu\kappa$, β is the coefficient of volumetric expansion, κ thermal diffusivity and ν kinematic viscosity.

Four thermocouples were positioned axially in the cylinder with their junctions at the mid-plane. These thermocouples could be traversed radially yielding temperature profiles along four diameters of the cross-section. These four diameters were at increments of 45° from the diameter containing the two heat exchangers.

The transparency of the working fluid made it suitable for photographic equipment to be employed. The scheme to trace streamlines consisted of introducing some small particles into the fluid. By tracing the path of a particle, a streamline could be determined since at steady state, streamlines and particle paths are identical. All measurements in this work were made after steady state had occurred. The particles were made from a Goodyear plastic known commercially as Pliolite, Type S-5 and ranged in size from 105 to 149 μ.

A camera with a 73 mm lens was used to focus axially through one of the end plates to the mid-plane of the cylinder. A viewer was placed at the picture-taking end of the camera so that the actual mid-plane could be viewed by eye. Inserts into the viewer were made from clear plastic. The scheme involved marking dots on the inserts at specific time intervals indicating the path of the particle. After the particle had made a complete loop, the streamline could be seen in its entirety.

Velocity changes along a streamline were found to be small enough so that one time interval, carefully chosen, would suffice for the entire streamline. This is desirable because it makes the relative velocity along the streamline obvious from the streamline trace alone, a greater spacing between dots indicating a greater velocity. However, the velocity change from streamline to streamline was quite considerable at times. This made it impossible for the same time interval to be used throughout the test.

It must be remembered that the particles observed could not be chosen with respect to any specific values of a stream function. Therefore, the streamline spacing presented does not correspond to equal increments of the stream function.

The photographic equipment used for the streamline traces was also used to obtain the velocity profiles. However, the viewer was removed from the camera and a Graflex Polaroid Back was attached in its place. Three exposures were taken on each film. Therefore, three dots appeared on each piece of film for each particle photographed. The interval between the first and second exposure was always twice as large as the interval between the second and third exposure. This was done so that the direction of travel of the particle was indicated as well as the magnitude of its velocity. Non-dimensional velocities are plotted. The non-dimensionalization scheme is explained by Ostrach (1968).

The streamline traces give much more information about the dynamics of the flow than do velocity profiles. However, velocity profiles were desired for comparison with previous experimental work. Therefore, only the velocity profiles needed for these comparisons were obtained.

Two starting conditions were investigated. First, the condition of starting from rest with a uniform temperature equal to the average of the two imposed temperatures was studied. Before the constant temperature circulators were activated, the fluid was allowed to remain at rest in the cylinder for 24 h. The temperature of the room was kept at T_0, so that when the circulators were activated, the temperature of the fluid was also T_0. Secondly, the steady-state results for one heating angle were used as the initial condition for another heating angle. At the end of a complete test in which the fluid was started from rest, the equipment was allowed to remain in operation while the entire cylinder was rotated to a new value of the heating angle. The cylinder was allowed to remain in its new position a minimum of 12 h before temperature profiles or streamline traces were attempted. However, the boundary condition was checked from time to time during this period. It was found that the boundary condition remained constant.

Specific tests were repeated to show the duplicability of the results. A complete run was also made in a plane axially displaced from the mid-plane of the cylinder by 1 in. These results compared with the corresponding readings at the mid-plane indicating a lack of axial change.

During the course of a streamline trace, it is possible to detect (but not measure) axial velocity. Axial motion would be present if a change of focus were required to keep the particle in view. Axial motion was found to be negligible in the results which are represented here. The lack of axial change, together with the lack of axial motion, ensure that, to the accuracy of these measurements, the experiment does indeed yield two-dimensional results. Furthermore, Sabzevari & Ostrach's (1966) results, using the same apparatus, agreed with the two-dimensional theory.

3. Results and conclusions

Steady-state results were obtained when the flow was started from rest for four values of the heating angle: 0°, 15°, 45° and 60°. Temperature profiles and streamline patterns are presented for each of these cases. Velocity profiles along the vertical and horizontal diameters are presented for heating angles of 0° and 45° only.

Figures 3–5 represent the streamline pattern, velocity profiles, and temperature profiles, respectively, when heating is directed from the side. From figure 3, it can be seen that the motion of the interior 'core' region consists of two small cells whose centres lie along the horizontal diameter. Both cells rotate counterclockwise, as is the motion of all the streamlines presented. From the distances and time intervals between points on the various streamlines, it can be seen that the velocities encountered on streamlines 4 and 5 are about an order of magnitude less than those encountered on streamlines 1 and 2. The shape of the streamlines should be noted in that, at a distance from the centre of the cylinder, they are elliptical-like in shape with the major axis being along the horizontal diameter. Streamline 2 in particular exhibits an almost horizontal shape except in the neighbourhood of $\theta = 0°$ and 180°. This would tend to agree with the prediction of Hantman & Ostrach (1968) of horizontal streamlines when heating is directed from the side. The velocity changes along a streamline can be detected by the relative spacing of the dots comprising the streamline. Since the time intervals between dots are constant for a streamline, greater spacing of the dots along a streamline is equivalent to a greater velocity. With this in mind, and with reference to streamlines 1–3, it can be seen that the flow is fastest in the areas near $\theta = 0°$ and 180°, and slows down in the areas near $\theta = 90°$ and 270°. The streamline pattern seems to be almost symmetrical about the horizontal diameter.

From the velocity profile along the horizontal diameter, as shown in figure 4, it can be seen that for values of r, defined as R/R_0, less than about 0·6 when $\theta = 0°$ or 180°, the velocity is very small, as much as an order of magnitude less than the maximum velocities encountered. (v_θ is the dimensionless tangential velocity $= R_0 V_0/\nu Gr$, where V_0 is the tangential velocity.) This is in agreement with the velocity analysis proposed from the streamline trace. The places where this curve crosses the axis, at either side of the centre of the cylinder, indicate the centres of rotation of the two small cells. This too agrees with the streamline trace. The shapes of the curves along both the horizontal and vertical diameters agree qualitatively with those experimentally found by Sabzevari & Ostrach (1966) when a cosine boundary condition was used. However, they did not detect the two small cells. Gill's (1966) work predicts a streamline pattern similar to the present one for the rectangular geometry heated from the side.

Figure 5 shows the temperature profiles along four diameters. It is obvious from the profile along the vertical diameter that in the central portion of the cylinder, the fluid is thermally stratified. The four curves are nearly straight lines in this region indicating constant temperature gradients, $\partial\tau/\partial r$. The gradient along the vertical diameter is largest and yields a value of $|\partial\tau/\partial r| \simeq 0{\cdot}473$. These temperature profiles too are near to symmetrical in the core region. The temperature at $r = 0$ is slightly above T_0, the average of the two imposed temperatures.

The streamline pattern and temperature profiles when $\phi = 15°$ are presented, respectively, in figures 6 and 7. This streamline pattern is very similar in shape to

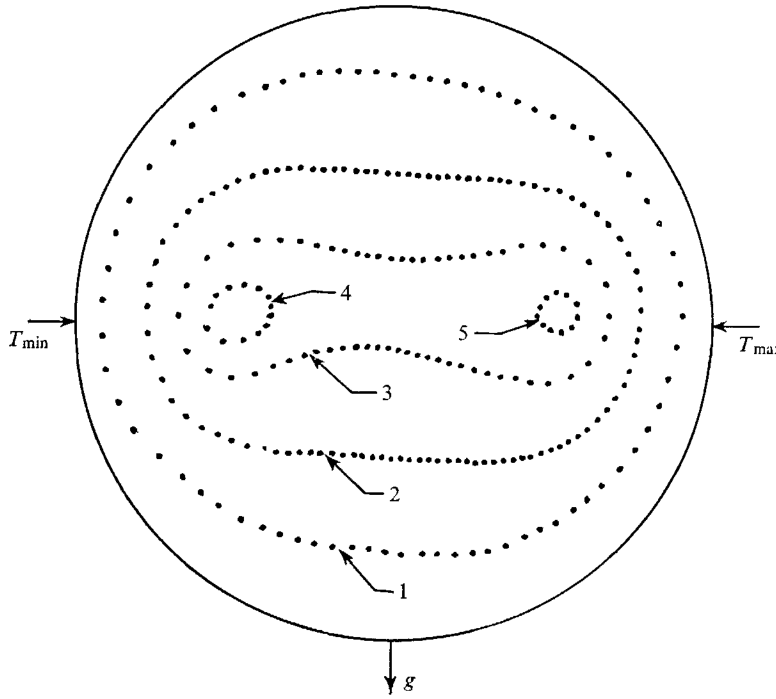

FIGURE 3. Streamline pattern for $\phi = 0°$ (from rest)

Streamline	Interval between points (min)	Total time (min)
1	1	71
2	1	98½
3	4	235
4	8	100
5	8	86

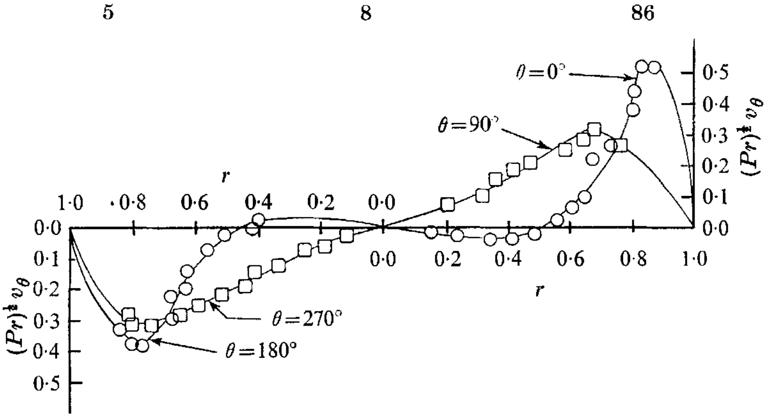

FIGURE 4. Velocity profiles for $\phi = 0°$ (from rest). ○, Horizontal diameter; □, vertical diameter.

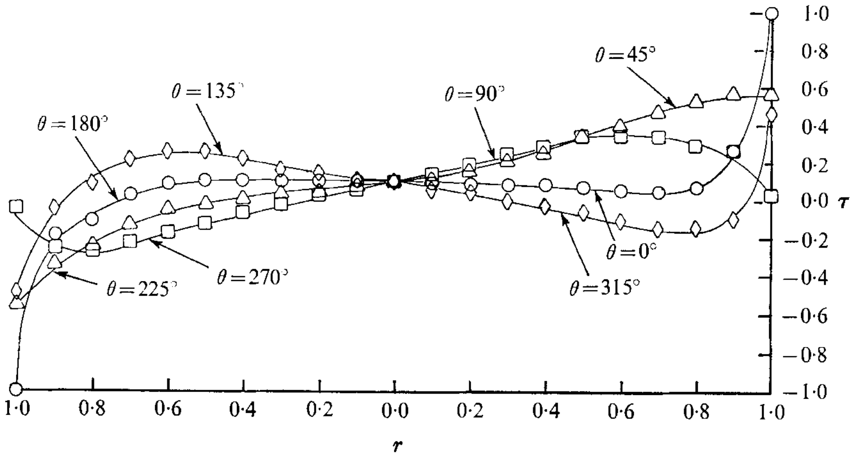

FIGURE 5. Temperature profiles for $\phi = 0°$ (from rest).

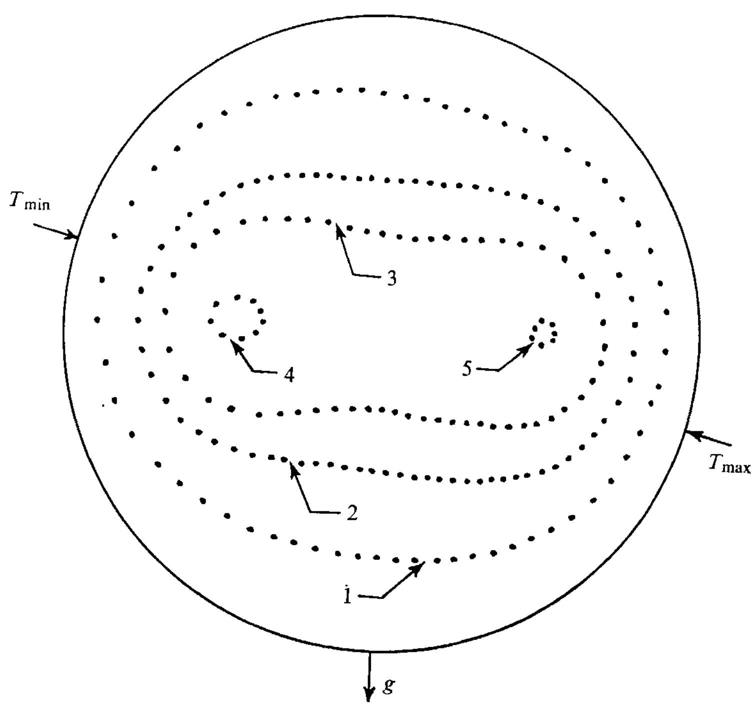

FIGURE 6. Streamline pattern for $\phi = 15°$ (from rest)

Streamline	Interval between points (min)	Total time (min)
1	1	$61\frac{1}{2}$
2	1	$73\frac{1}{4}$
3	2	107
4	8	70
5	8	52

the streamline pattern when $\phi = 0°$. However, it should be noted that the velocities for $\phi = 15°$ are greater than those for $\phi = 0°$ when compared at the same locations. The temperature profiles are also similar, but the degree of stratification is less for $\phi = 15°$ than it was for $\phi = 0°$. Along the $\theta = 75$–$255°$ diameter, where the largest gradient is experimentally encountered, $|\partial\tau/\partial r| \simeq 0{\cdot}362$ when $\phi = 15°$ as compared to 0·473 along the vertical diameter when $\phi = 0°$. To obtain a better comparison of the degree of stratification for the two cases, the gradient for $\phi = 15°$ is divided by cos 15° to get a 'vertical diameter' value of $|\partial\tau/\partial r| \simeq 0{\cdot}374$. This is still well below the value encountered for $\phi = 0°$. The temperature profile along the vertical diameter could not be measured directly because of the way in which the cylinder was constructed.

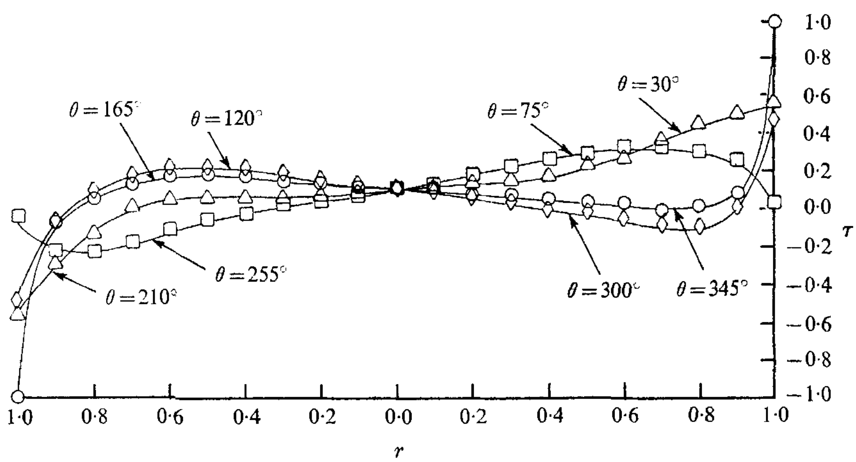

FIGURE 7. Temperature profiles for $\phi = 15°$ (from rest).

Figure 8 represents the streamline pattern when $\phi = 45°$. This pattern is markedly different from the patterns for $\phi = 0°$ and $\phi = 15°$ in that the motion consists entirely of one cell. The pattern is unsymmetric with the cell being centred to the left of the centre of the cylinder. This streamline pattern seems to be similar to that found by Somerville (1967) for a rectangular cross-section unevenly heated from below. However, in Somerville's work the cell was centred on the warm side of the cross-section, while in this study the cell is centred towards the cold side. The area to the right of streamline 4 was investigated in search of another cell, but none could be found. The movement of the particles there was such as to circumscribe streamline 4. The streamlines are elliptical-like in shape again, but now the major axis has a slightly negative slope. This negative slope may be explained on the basis of physical reasoning. The driving force for the fluid motion is the density differences in the fluid caused by the thermal boundary condition. The extrema in the thermal boundary condition are located at either end of a diameter of the cylinder. As the fluid passes these extrema the driving force is most influential. The driving force, however, acts in the vertical direction. Therefore the fluid would tend towards vertical motion in the vicinity of the extrema, vertically upwards near T_{max} and vertically downwards near T_{min}. This factor together with the physical constraint of the cylinder walls would cause the major axis to be at an angle intermediate between the horizontal and the value of the heating angle. Ostrach & Menold predicted elliptical-like streamlines when

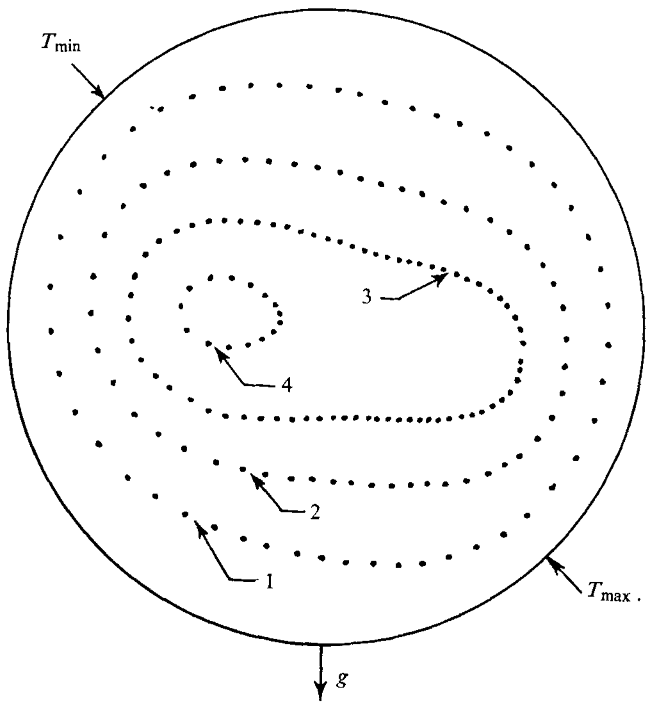

FIGURE 8. Streamline pattern for $\phi = 45°$ (from rest)

Streamline	Interval between points (min)	Total time (min)
1	1	50
2	1	51
3	1	$68\frac{1}{2}$
4	4	60

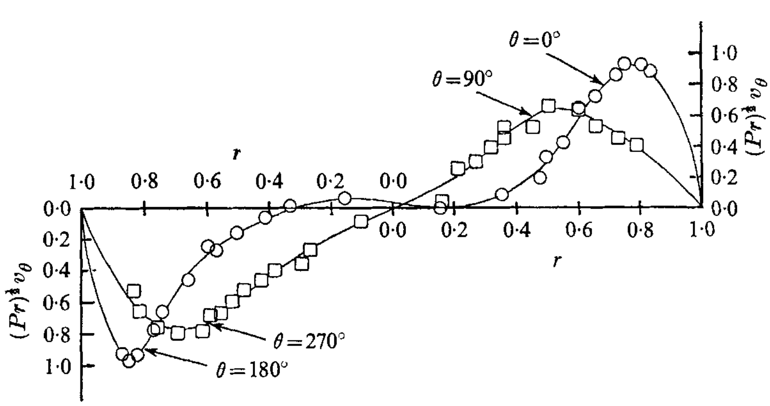

FIGURE 9. Velocity profiles for $\phi = 45°$ (from rest). ○, Horizontal diameter; □, vertical diameter.

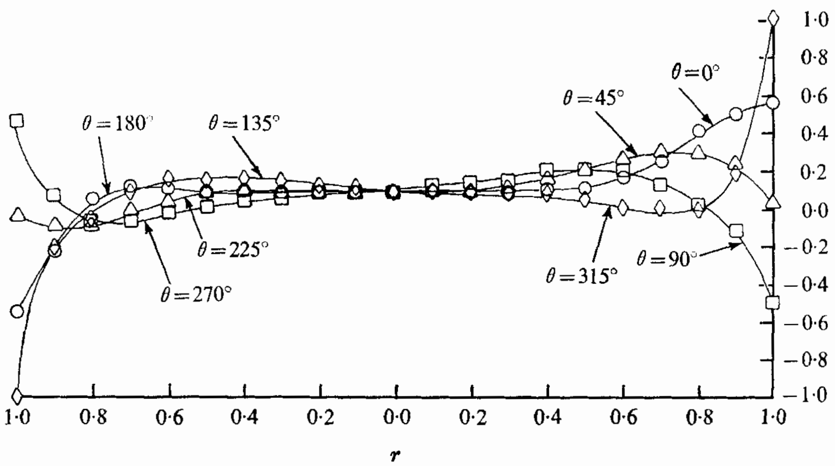

FIGURE 10. Temperature profiles for $\phi = 45°$ (from rest).

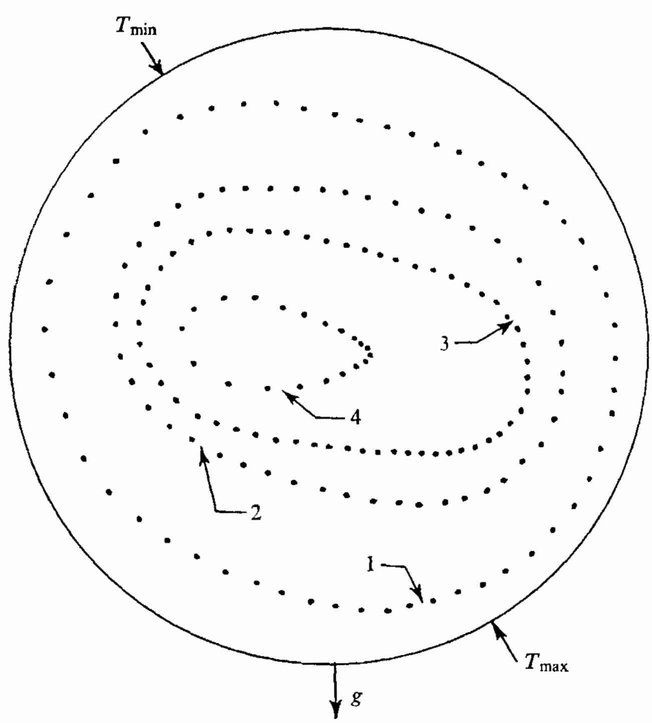

FIGURE 11. Streamline pattern for $\phi = 60°$ (from rest)

Streamline	Interval between points (min)	Total time (min)
1	1	$52\frac{1}{2}$
2	1	$45\frac{1}{2}$
3	1	$54\frac{1}{2}$
4	4	79

$\phi = 45°$ with the major axis having a positive slope. Note also that the fluid in the core is moving faster now, than it was when $\phi = 0°$ or $15°$. Figure 9 shows the velocity profiles along the vertical and horizontal diameters when $\phi = 45°$. The profiles agree with the streamline trace of figure 8. Both curves agree qualitatively with those of Sabzevari & Ostrach (1966). Again, Sabzevari & Ostrach show the velocity to be zero near the centre of the horizontal diameter where small velocities were found in this study. Figure 10 shows the temperature profiles when $\phi = 45°$. There is a temperature gradient present but it is much smaller than that when $\phi = 0°$ or $15°$. The maximum gradient, along the vertical diameter, is only 0·225 now as compared to 0·374 for $\phi = 15°$ and 0·473 for $\phi = 0°$.

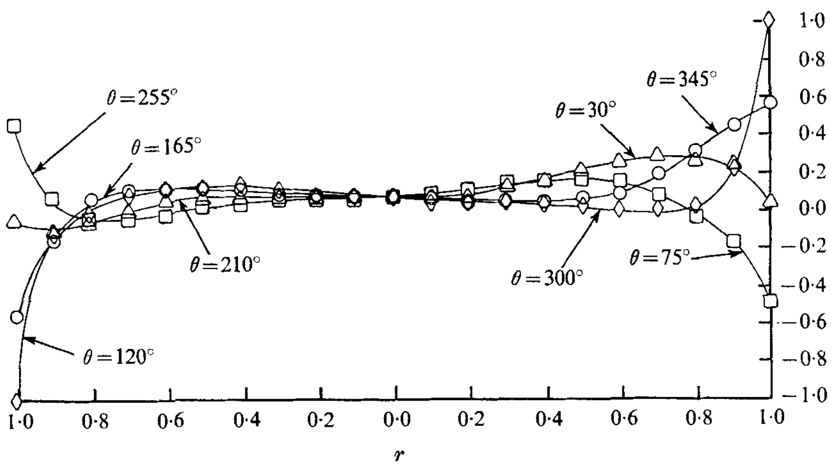

FIGURE 12. Temperature profiles for $\phi = 60°$ (from rest).

The streamline pattern when $\phi = 60°$, shown in figure 11, is quite similar to that when $\phi = 45°$, except that the fluid is moving slightly faster now. The single-cell and unsymmetric shape of the streamlines is about the same as when $\phi = 45°$. The slope of the major axis of the streamlines is slightly more pronounced than it was when $\phi = 45°$. The temperature profiles for $\phi = 60°$, shown in figure 12, depict even less stratification than when $\phi = 45°$. The maximum measured gradient, along the $\theta = 75–255°$ diameter is only 0·141. This yields a 'vertical diameter' value of 0·146.

Several trends can be observed from the preceding results. When the heating angle is zero, the motion of the core region consists of two small slowly rotating cells situated along the horizontal diameter. The motion of the remainder of the fluid is such as to circumscribe both of these small cells. This causes the velocity to be nearly horizontal above and below the cells, with a motion towards the left above the cells and towards the right below them. The velocity of the fluid in these cells is about an order of magnitude less than the velocities encountered further from the centre of the cylinder. Also, the fluid in the central region is thermally stratified, with a relatively large temperature gradient, and with the temperature increasing vertically upward. As ϕ is increased from zero, this behaviour continues but the velocities increase as the temperature gradients decrease. At some value of ϕ, intermediate between $\phi = 15°$ and $\phi = 45°$, the two-cell behaviour ceases, and one cell, centred to the left of the centre of the cylinder, appears. The streamline pattern is much less symmetric than before. The fluid in the core is still thermally stratified, but the temperature gradient is a fraction of what it was for the smaller values of ϕ. As ϕ is increased further, the one-cell behaviour persists with the velocities continuing to increase and the temperature gradient continuing to decrease. The core region seems to be approaching an isothermal rotating state as ϕ approaches 90°.

Tests were run at values of ϕ greater than 60°, but if ϕ became too large, axial motion was observed. For $\phi = 67\frac{1}{2}°$, the two-dimensional model started to break down. For $\phi = 75°$, there was too much axial motion to make any meaningful measurements. Ostrach & Pnueli (1963) showed that when heating is directed from below, a thermal instability exists and flow always starts in the direction of the longest dimension. Axial motion will dominate as long as the length of the cylinder is greater than its diameter. Since this is the case here, a two-dimensional situation is meaningless when heating is directed from below. It appears that as the heating angle increases past 60°, this effect becomes important.

Ostrach & Menold pointed out that the maximum temperature of the fluid core is out of phase with the maximum temperature on the boundary. This study bears this out, since the maximum fluid temperature in the core is always above the horizontal diameter, while the maximum temperature on the boundary is on or below the horizontal diameter.

It cannot be overstressed that the streamline spacing presented for the various streamline patterns has nothing whatever to do with a stream function. For the patterns at $\phi = 0°$ and 15° in particular, the difference in the size of the streamlines showing the presence of the two cells should not be considered important since these two streamlines are not necessarily at the same value of a stream function. They merely show the existence of the cells.

In all the cases presented, some unsymmetric behaviour was encountered. This is principally attributed to two factors. First, the viscosity-temperature relation for the working fluid, as provided by the General Electric Company, shows that the viscosity change will be about 10 % over the temperature range indicative of this experiment. Secondly, the thermal boundary condition, which is the driving force for the fluid motion, is slightly unsymmetric.

It was now desired to see if the initial condition would have an effect on the steady-state solution. Two different steady-state types of flow were previously encountered: the two-cell, highly stratified, slower flow indicative of small heating angles; and the one-cell, slightly stratified, faster moving flow indicative of greater heating angles. The cylinder was now allowed to reach steady state with $\phi = 45°$. Then the cylinder was rotated so $\phi = 0°$ and again allowed to reach steady state. In this way the steady-state results for $\phi = 45°$, as depicted in figures 8 and 10 became the initial conditions for $\phi = 0°$. The new steady-state results are shown in figures 13 and 14. Figure 13 shows that the two-cell mode has again appeared and that the velocities have decreased. Figure 14 shows that the degree of thermal stratification has increased. The gradient along the vertical diameter is now 0·467 as compared to 0·473 for $\phi = 0°$ starting from rest and 0·225 for $\phi = 45°$. It is concluded that the flow depicted in figures 13 and 14 is the same as that depicted in figures 3 and 5. In other words, the steady-state results for heating from the side are the same whether the flow was started from rest or from the state depicted in figures 8 and 10. Certainly, the initial condition did not persist at $\phi = 0°$.

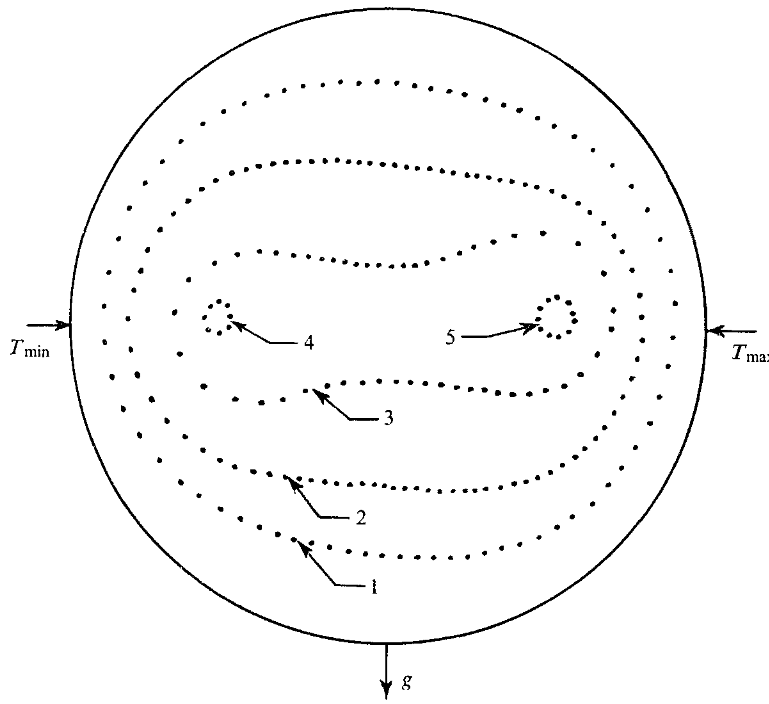

FIGURE 13. Streamline pattern for $\phi = 0°$ (started from $\phi = 45°$)

Streamline	Interval between points (min)	Total time (min)
1	1	71
2	1	88
3	4	206
4	8	64
5	8	82

In much of the previous work, the core is referred to as either stagnant and thermally stratified or as isothermal and rotating. In this work, some motion in the core was always observed and some degree of thermal stratification was always detected. If the results for $\phi = 45°$ were to be interpreted as a nearly-isothermal rotating core, it is seen that this situation does not remain at $\phi = 0°$. It is still possible that other initial conditions will cause other steady-state results, but that is a task for future investigations. In this work, only these two initial conditions were investigated.

The results presented here agree qualitatively with other experimental work. Other works do not present experimentally found streamlines, but velocity and temperature profiles are presented. Martini & Churchill (1960), using the step function boundary condition, conclude that there is a narrow ring of circulating fluid when heating is directed from the side. This ring is narrowest near $\theta = 0°$ and 180° and increases in thickness near $\theta = 90°$ and 270°. Also the velocities are largest near $\theta = 0°$ and 180° and decrease as the fluid approaches $\theta = 90°$ and 270°. This behaviour is indicative of the streamlines presented in this study. Martini & Churchill conclude that the core region is thermally stratified and essentially stagnant but has slow-moving eddies. Again, the results here bear this out.

The results of Sabzevari & Ostrach (1966) agree to a certain extent with the present results, although they used a cosine thermal boundary condition. For heating from the side, their velocity profiles agree qualitatively with the ones presented here, thus indicating a core region of the same general shape as was found here. However, they found no evidence of two cells. They do agree that the fluid is thermally stratified. At $\phi = 15°$, Sabzevari & Ostrach find evidence of

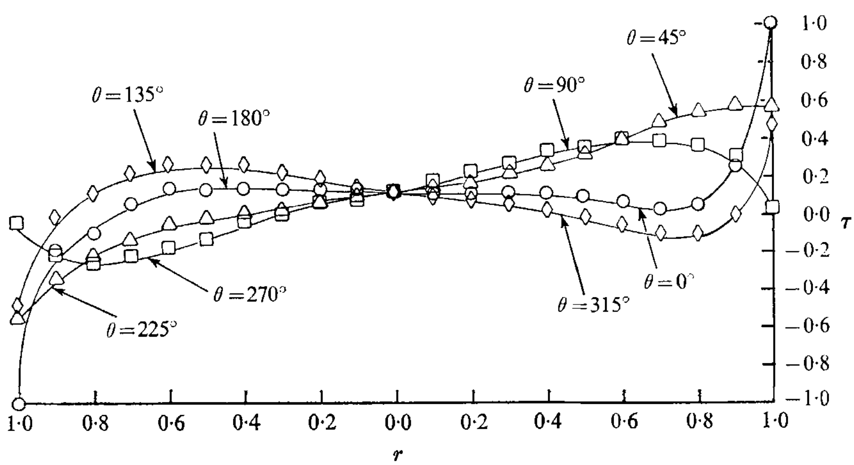

FIGURE 14. Temperature profiles for $\phi = 0°$ (started from $\phi = 45°$).

the two cells, but do not find them to be situated along the horizontal diameter For $\phi = 45°$ and 60°, they conclude that the core region is isothermal and rotating. The present work also found the core to be rotating but did detect a small temperature gradient. The two works agree in that the types of flow are different for large values of ϕ as opposed to small values of ϕ.

4. Summary

For small heating angles, the core region consists of two small, slowly moving cells situated along the horizontal diameter. The remainder of the fluid rotates around both of these cells such that the velocities above and below them are nearly horizontal. The core region is thermally stratified with a relatively large temperature gradient.

For large heating angles, the motion consists of one cell situated to the left (cold side) of the centre of the cylinder. The core region is thermally stratified, but the temperature gradient is very small.

Over the entire range of heating angles, there is a trend for the velocity to increase in the entire cross-section and temperature gradient to decrease in the core, as the heating angle is increased.

The steady-state results, when heating is directed from the side, are the same whether the motion was started from rest or from the nearly-isothermal rotating core results indicative of heating from 45°.

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